Coupled channels calculation of a $\pi\Lambda N$ quasibound state

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We extend the study of a $J^P = 2^+, I = \frac{3}{2}$ $\pi\Lambda N$ quasibound state [Phys. Rev. D **78**, 014013 (2008)] by solving nonrelativistic Faddeev equations, using ${}^3S_1 - {}^3D_1$, $\Lambda N - \Sigma N$ coupled channels Chiral Quark Model local interactions, and πN and coupled $\pi\Lambda - \pi\Sigma$ separable interactions fitted to the position and decay parameters of the $\Delta(1232)$ and $\Sigma(1385)$ resonances, respectively. The results exhibit a strong sensitivity to the p-wave pion-hyperon interaction, with a $\pi\Lambda N$ quasibound state persisting over a wide range of acceptable parametrizations.

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I. INTRODUCTION

The success of the nonrelativistic quark model (QM) during the 1970s in reproducing the SU(3) octet and decuplet baryon masses in terms of $3q$ configurations was followed by QM studies of $6q$ configurations that aimed particularly at elucidating the baryon-baryon short-range dynamics and making related predictions for dibaryon bound or quasibound states. It is remarkable that decades of experimental searches for dibaryons have so far yielded no unambiguous evidence for a dibaryon state. In the nonstrange sector, where the quark cluster calculations for $L = 0$ $6q$ configurations [1] suggest only a weakly bound $\Delta\Delta$ dibaryon with $(J^P, I) = (3^+, 0)$, there is a recent indication from $np \to d\pi\pi$ reactions at CELSIUS-WASA for a resonance structure at $M_R \approx 2.36$ GeV and $\Gamma_R \approx 80$ MeV that might suggest a $\Delta\Delta$ dibaryon bound by about 100 MeV, but still about 200 MeV above the $d\pi\pi$ threshold [2]. In the strange sector, Jaffe's dibaryon H [3] with strangeness $S = -2$ and quantum numbers $(J^P, I) = (0^+, 0)$ which was predicted as a genuinely bound state well below the $\Lambda\Lambda$ threshold, perhaps the most cited ever prediction made for any dibaryon, has not been confirmed experimentally to date in spite of several extensive searches [4]. Another equally ambitious early prediction was made by Goldman *et al.* [5], also using a variant of the MIT bag model, for $(J^P; I) = (1^+, 2^+; \frac{1}{2})$ $S = -3$ dibaryons dominated by ΩN structure and lying below the $\Xi\Lambda$ threshold. More realistic quark cluster calculations by Oka *et al.* [6], applying resonating group techniques, did not confirm Jaffe's deeply bound H, placing it just below the ΞN threshold as a resonance about 26 MeV above the $\Lambda\Lambda$ threshold. The underlying binding mechanism common to all of these orbital angular momentum $L = 0$ configurations is the dominance of the color-magnetic interaction for gluon exchange between quarks, a feature emphasized by Oka [7] who systematically studied $L = 0$ dibaryon configurations

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that may benefit from a short-range attraction. Following earlier quark cluster calculations [1, 6], these calculations resulted in no strange dibaryon bound states, and for the ΩN-dominated $S = -3$ bound-state configurations predicted in Ref. [5], in particular, only a $(J^P, I) = (2^+, \frac{1}{2})$ quasibound state resulted.

For strangeness $S = -1$, which is the focus of the present work, no $L = 0$ dibaryons have been suggested for the lowest energy $I = \frac{1}{2}$ $\Lambda N - \Sigma N$ coupled channels, where the long-range pion exchange interaction is dominant, particularly for the ${}^3S_1 - {}^3D_1$ system through the tensor component. Although old $K^- d \to \pi^- \Lambda p$ data [8] had suggested resonant Λp structures at the ΣN threshold and 10 MeV above it, a $(J^P, I) = (1^+, \frac{1}{2})$ ΣN quasibound state is not necessarily required in order to reproduce the general shape of the Λp spectrum as shown by multichannel Faddeev calculations [9, 10]. Several low-lying $L = 1$ ΛN resonances were predicted in singlet and triplet configurations in a QM study by Mulders *et al.* [11], but negative results, particularly for the singlet resonance, were reported in dedicated K^- initiated experiments [12, 13] near the ΣN threshold. At higher energies, Oka's analysis [7] drew attention to a $(J^P, I) = (2^+, \frac{1}{2})$ dibaryon predominantly of a $\Sigma(1385)N - \Sigma\Delta(1232)$ coupled channels structure resonating about the $\Sigma\Delta(1232)$ threshold, approximately 100 MeV above the lower $\Sigma(1385)N$ threshold. We note that these two channels are substantially higher in mass, by about 300 MeV, than the $S = -1$ thresholds of ΛN and ΣN.

In a recent paper [14] we studied within three-body Faddeev calculations the possible existence of a $\pi\Lambda N$ quasibound state, driven by the two-body $({}^{2S+1}L_J, I) = ({}^2P_{\frac{3}{2}}, \frac{3}{2})$ πN resonance $\Delta(1232)$ and the $({}^2P_{\frac{3}{2}}, 1)$ $\pi\Lambda$ resonance $\Sigma(1385)$, for a ΛN $({}^3S_1, \frac{1}{2})$ configuration, all of which were represented by means of single-channel separable potentials. The coupling to the pionless ΣN channel, with threshold about 60 MeV below the $\pi\Lambda N$ threshold, was disregarded. It was felt that this coupling was mostly responsible for the width of the $\pi\Lambda N$ quasibound state. The three-body channel $(J^P, I) = (2^+, \frac{3}{2})$ was selected since all the angular momenta, spins, and isospins in this channel have maximum values and, therefore, it is likely to benefit from maximal attraction of both $\Delta(1232)$ and $\Sigma(1385)$ resonances. This opportunity is unique to strange and charmed systems: a similar choice of $(J^P, I) = (2^+, 2)$ for πNN, with each πN pair interacting in the $(\frac{3}{2}^+, \frac{3}{2})$ resonating channel, implies a $(1^+, 1)$ Pauli-forbidden NN configuration. In terms of dibaryons, the $\pi\Lambda N$ $(J^P, I) = (2^+, \frac{3}{2})$ quasibound state is a deeply bound $\Sigma(1385)N - \Lambda\Delta(1232)$ $L = 0$ dibaryon, at energy considerably below the $(J^P, I) = (2^+, \frac{1}{2})$ $\Sigma(1385)N - \Sigma\Delta(1232)$ $L = 0$ dibaryon suggested by the quark cluster model of Ref. [7].

Whereas the interactions in the pion-baryon resonating channels in first approximation are adequately represented by rank-one attractive separable potentials, the baryon-baryon interaction requires a rank-two separable potential to simulate both the attraction and repulsion that meson-exchange models normally yield. Indeed, we found a strong dependence of the calculated $\pi\Lambda N$ binding energy on the balance between repulsion and attraction in the ${}^3S_1$ ΛN channel [14]. It is therefore suggestive to consider a more realistic hyperon-nucleon interaction in the $J^P = 1^+$ channel. In the present work we used the hyperon-nucleon (YN) Chiral Quark Model (CQM) interaction described in Refs. [15, 16] in terms of ${}^3S_1 - {}^3D_1$, $\Lambda N - \Sigma N$ coupled channels *local* potentials. For consistency, we also generalized our previous single-channel model of $\Sigma(1385)$ as a $\pi\Lambda$ resonance to a family of pion-hyperon (πY) interaction models, in terms of $\pi\Lambda - \pi\Sigma$ coupled channels separable interactions fitted to the position, width and decay branching ratios of $\Sigma(1385)$. Furthermore, we studied the dependence of the calculated $\pi\Lambda N$ binding energy on the πY interaction.

In our previous work [14], based on separable potentials, we considered both a nonrelativistic and a relativistic three-body formalism from which we deduced that the nonrelativistic results do not change much when the relativistic

formalism is used instead. This is relevant for the validity of the results of the present work which are based on the hyperon-nucleon interaction derived from the CQM within a *nonrelativistic* formalism. Therefore, in the present calculation we consider only a nonrelativistic framework. The results of the present three-body Faddeev calculations leave wide room for the existence of a $(2^+, \frac{3}{2})$ $\pi\Lambda N$ quasibound state indicating, however, a strong dependence on the short-range behavior of the least known πY and YN two-body subsystems.

The plan of the paper is as follows. In Sec. II we describe the two-body interactions in the pion-nucleon, pion-hyperon and hyperon-nucleon subsystems. In Sec. III we derive the Faddeev equations of the pion-nucleon-hyperon system. Finally, we discuss our results in Sec. IV and summarize the work in Sec. V.

II. THE TWO-BODY SUBSYSTEMS

We will denote the hyperon, nucleon, and pion as particles 1, 2, and 3, respectively, and refer to the two-body subsystems by a subscript for the spectator particle. Thus, pion-nucleon is subsystem 1, pion-hyperon is subsystem 2, and hyperon-nucleon is subsystem 3. The conventional reduced masses are given by

$$\eta_i^\alpha = \frac{m_j m_k}{m_j + m_k}, \qquad \nu_i^\alpha = \frac{m_i(m_j + m_k)}{m_i + m_j + m_k}, \tag{1}$$

where a superscript $\alpha = \Lambda, \Sigma$ has been added to indicate whether particle 1 is a Λ or a Σ hyperon and, obviously,

$$\eta_1^\Lambda \equiv \eta_1^\Sigma \equiv \eta_1 = \frac{m_\pi m_N}{m_\pi + m_N}. \tag{2}$$

However, an average hyperon mass

$$m_Y = \frac{m_\Lambda + m_\Sigma}{2} \tag{3}$$

was used in the following reduced masses:

$$\nu_2 = \frac{m_N(m_\pi + m_Y)}{m_N + m_\pi + m_Y}, \qquad \nu_3 = \frac{m_\pi(m_N + m_Y)}{m_\pi + m_N + m_Y}. \tag{4}$$

The πY and YN amplitudes are given by 2×2 matrices, to account for the coupling between $\pi\Lambda$ and $\pi\Sigma$ and between ΛN and ΣN, respectively. The πN amplitude in the three-body system is also given by 2×2 matrix, since the energy dependence of the two-body subsystem depends on whether the spectator particle is a Λ or a Σ.

A. The pion-nucleon subsystem

Since the πN subsystem is dominated by the $\Delta(1232)$ resonance, a rank-one separable interaction is considered sufficient:

$$< p_1|V_1|p_1' >= \gamma_1 g_1(p_1) g_1(p_1') \ , \tag{5}$$

so that the corresponding two-body t-matrix is given by

$$< p_1|t_1(E)|p_1' >= g_1(p_1)\tau_1(E) g_1(p_1') \ , \tag{6}$$

where $E = p_0^2/2\eta_1$ with p_0 the correct relativistic πN center of mass (c.m.) momentum and

$$\tau_1^{-1}(E) = 1/\gamma_1 - \int_0^\infty p_1^2 dp_1 \frac{g_1^2(p_1)}{E - p_1^2/2\eta_1 + i\epsilon} \ . \tag{7}$$

The form factor $g_1(p_1)$ was obtained from a very good fit of the P_{33} phase shift [17] for $0 \leq T_{\rm lab} \leq 250$ MeV in the form

$$g_1(p_1) = p_1[e^{-p_1^2/\beta_1^2} + A_1 p_1^2 e^{-p_1^2/\alpha_1^2}] \ , \tag{8}$$

with $\gamma_1 = -0.03317$ fm^4, $A_1 = 0.2$ fm^2, $\beta_1 = 1.31$ fm^{-1}, and $\alpha_1 = 3.2112$ fm^{-1}.

In the three-body calculation, when the πN subsystem is embedded in the πYN system, the energy argument of the isobar propagator $\tau_1(E)$ depends on whether the spectator hyperon is a Λ or a Σ, so that the separable πN amplitude (6) takes the form

$$t_1 = |g_1> \begin{pmatrix} \tau_\Lambda(q_1) & 0 \\ 0 & \tau_\Sigma(q_1) \end{pmatrix} < g_1|, \tag{9}$$

where

$$\tau_\alpha(q_1) = \tau_1(E - \delta_{\alpha\Sigma}\Delta E - q_1^2/2\nu_1^\alpha); \qquad \alpha = \Lambda, \Sigma, \tag{10}$$

with q_1 the relative momentum between the hyperon and the πN subsystem and

$$\Delta E = m_\Sigma - m_\Lambda. \tag{11}$$

B. The pion-hyperon subsystem

The πY subsystem is dominated by the $\Sigma(1385)$ p-wave resonance which decays mainly into $\pi\Lambda$ and $\pi\Sigma$ with branching ratios of $(87.0 \pm 1.5)\%$ and $(11.7 \pm 1.5)\%$, respectively [18]. To account for the coupling $\pi\Lambda - \pi\Sigma$, we assume a coupled channels separable interaction:

$$< p_2|V_2^{\alpha\beta}|p_2' >= \gamma_2 g_2^\alpha(p_2) g_2^\beta(p_2'); \qquad \alpha, \beta = \Lambda, \Sigma, \tag{12}$$

so that the corresponding two-body t-matrix is given by

$$< p_2|t_2(E)|p_2' >= g_2^\alpha(p_2)\tau_2(E) g_2^\beta(p_2'); \qquad \alpha, \beta = \Lambda, \Sigma, \tag{13}$$

with

$$\tau_2^{-1}(E) = 1/\gamma_2 - \int_0^\infty p_2^2 dp_2 \frac{[g_2^\Lambda(p_2)]^2}{E - p_2^2/2\eta_2^\Lambda + i\epsilon} - \int_0^\infty p_2^2 dp_2 \frac{[g_2^\Sigma(p_2)]^2}{E - \Delta E - p_2^2/2\eta_2^\Sigma + i\epsilon}. \tag{14}$$

Again, $E = p_0^2/2\eta_2^\Lambda$ where p_0 is the correct relativistic $\pi\Lambda$ c.m. momentum and ΔE is chosen such that the $\pi\Lambda$ momentum at the $\pi\Sigma$ threshold has its correct value, that is

$$\Delta E = \frac{[(m_\Sigma + m_\pi)^2 - (m_\Lambda + m_\pi)^2][(m_\Sigma + m_\pi)^2 - (m_\Lambda - m_\pi)^2]}{8\eta_2^\Lambda (m_\Sigma + m_\pi)^2}. \tag{15}$$

TABLE I: Five choices (A-E) of form factor parameters for the coupled channels πY subsystem, Eqs. (12) and (18). The last line lists a single-channel $\pi\Lambda$ sixth model (F) with $c_2 = 0$. The last column lists values of the r.m.s. momentum [see Eq. (19)].

Model	A_2 (fm^2)	α_2 (fm^{-1})	γ_2 (fm^4)	c_2	$\sqrt{< p_2^2 >_{g_2}}$ (fm^{-1})
A	0.8	2.41372	-0.00604931	0.890227	4.11
B	1.0	2.29039	-0.00552272	0.925591	3.91
C	1.2	2.20024	-0.00501334	0.956818	3.76
D	1.5	2.10192	-0.00433208	0.997300	3.60
E	1.8	2.03076	-0.00375829	1.03166	3.48
F	3.21	2.20024	-0.00149204	0	3.79

The πY t-matrix (13) in the πYN system may be written in compact notation as a 2×2 matrix

$$t_2 = \begin{pmatrix} |g_2^\Lambda > \\ |g_2^\Sigma > \end{pmatrix} \tau_N(q_2) \left(< g_2^\Lambda| \;\; < g_2^\Sigma| \right), \tag{16}$$

where

$$\tau_N(q_2) = \tau_2(E - q_2^2/2\nu_2). \tag{17}$$

The form factors $g_2^Y(p_2)$ of the separable πY p-wave potentials were taken in the form

$$g_2^\Lambda(p_2) = p_2(1 + A_2 p_2^2)e^{-p_2^2/\alpha_2^2}, \qquad g_2^\Sigma(p_2) = c_2 g_2^\Lambda(p_2). \tag{18}$$

Solutions exist for all values of A_2 between 0 and ∞. Therefore, in order to fit the position, width and decay branching ratios of $\Sigma(1385)$, we have at our disposal four free parameters: A_2, α_2, γ_2 and c_2, which provide for varying one of these while adjusting the other three to the three pieces of data. We thus constructed five models (models A-E) by considering five values of the parameter A_2, as shown in Table I. We also constructed a sixth model (model F) which shares the same range parameter α_2 with model C but which neglects the coupling to the $\pi\Sigma$ channel ($c_2 = 0$), as was done in our previous calculation [14]. It is instructive to classify the various πY interaction form factors $g_2^Y(p_2)$ according to their root-mean-square (r.m.s.) momentum, using the following expression for the mean-square momentum $< p_2^2 >_{g_2}$:

$$< p_2^2 >_{g_2} \;=\; \frac{\int_0^\infty g_2(p_2)\; p_2^2 \; d^3p_2}{\int_0^\infty g_2(p_2)\; d^3p_2} \;=\; 3\alpha_2^2 \frac{A_2\alpha_2^2 + \frac{1}{3}}{A_2\alpha_2^2 + \frac{1}{2}} \;\approx\; 3\alpha_2^2, \tag{19}$$

where the approximation owes to $2A_2\alpha_2^2 >> 1$. The resulting values of the r.m.s. momentum, listed in the last column of Table I, are close to $\sqrt{< p_2^2 >_{g_2}} \approx 3.8$ fm$^{-1} \approx 750$ MeV/c. For comparison, $\sqrt{< p_1^2 >_{g_1}} = 5.55$ fm$^{-1} \approx 1100$ MeV/c for the πN form factor $g_1(p_1)$ of Eq. (8).[1]

[1] This high-momentum value for g_1 does not rule out a spatial size of order 1 fm for $\Delta(1232)$. Indeed, if $\tilde{g}_1(r_1)$ is the Fourier transform of $g_1(p_1)$, for $\ell = 1$, then $\sqrt{< r_1^2 >_{\tilde{g}_1}} = 0.875$ fm.

C. The hyperon-nucleon subsystem

The YN interaction derived from the chiral quark model is a local potential obtained by application of the Born-Oppenheimer approximation to the chiral quark-quark interaction (consisting of confinement, one-gluon exchange, pseudovector-meson exchange, and scalar-meson exchange) with a fully antisymmetrized six-quark wave function [15, 16, 19]. In the case of the $J^P = 1^+$, $I = \frac{1}{2}$ channel, it leads to the following system of coupled equations:

$$\begin{aligned} t^{\alpha\beta}_{\ell\ell''}(p_3, p''_3; E) = \quad & V^{\alpha\beta}_{\ell\ell''}(p_3, p''_3) + \sum_{\gamma=\Lambda,\Sigma} \sum_{\ell'=0,2} \int_0^\infty {p'_3}^2 dp'_3 V^{\alpha\gamma}_{\ell\ell'}(p_3, p'_3) \\ & \times \frac{1}{E - \delta_{\gamma\Sigma}\Delta E - {p'_3}^2/2\eta_3^\gamma + i\epsilon} t^{\gamma\beta}_{\ell'\ell''}(p'_3, p''_3; E) : \qquad \alpha, \beta = \Lambda, \Sigma, \end{aligned} \tag{20}$$

with $\alpha, \beta = \Lambda, \Sigma$, $\ell, \ell'' = 0, 2$ and $E = p_0^2/2\eta_3^\Lambda$, where p_0 is the correct relativistic ΛN c.m. momentum, and ΔE is chosen such that the ΛN momentum at the ΣN threshold has its correct value, that is

$$\Delta E = \frac{[(m_\Sigma + m_N)^2 - (m_\Lambda + m_N)^2][(m_\Sigma + m_N)^2 - (m_\Lambda - m_N)^2]}{8\eta_3^\Lambda (m_\Sigma + m_N)^2}. \tag{21}$$

The YN t-matrix (20) may be written in compact notation as a 2×2 matrix

$$t_3 = \begin{pmatrix} t^{\Lambda\Lambda} & t^{\Lambda\Sigma} \\ t^{\Sigma\Lambda} & t^{\Sigma\Sigma} \end{pmatrix}, \tag{22}$$

where each YN t-matrix $t^{\alpha\beta}$ includes, in addition, a coupling between S ($\ell = 0$) and D ($\ell = 2$) waves.

III. THE THREE-BODY EQUATIONS

The Faddeev equations for the bound-state problem

$$T_i = \sum_{j \neq i} t_i G_0 T_j; \qquad i, j = 1, 2, 3, \tag{23}$$

couple the amplitudes T_1, T_2 and T_3 together. Eliminating the amplitude T_3 in favor of T_1 and T_2, one obtains

$$T_1 = t_1 G_0 t_3 G_0 T_1 + (t_1 + t_1 G_0 t_3) G_0 T_2, \tag{24}$$

$$T_2 = t_2 G_0 t_3 G_0 T_2 + (t_2 + t_2 G_0 t_3) G_0 T_1, \tag{25}$$

where, in order to allow for the $Y = (\Lambda, \Sigma)$ specification, one has

$$G_0 = \begin{pmatrix} G_0^\Lambda & 0 \\ 0 & G_0^\Sigma \end{pmatrix}, \tag{26}$$

Since the two-body amplitudes t_1 and t_2 are separable [see Eqs. (9) and (16)], the three-body amplitudes T_1 and T_2 are of the form

$$T_1 = |g_1> \begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix}, \tag{27}$$

$$T_2 = \begin{pmatrix} |g_2^\Lambda > \\ |g_2^\Sigma > \end{pmatrix} X_N, \tag{28}$$

where the subscript of the amplitude X indicates which particle is the spectator. Substitution of (27) and (28) into (24) and (25) leads to

$$\begin{aligned} \begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix} = & \begin{pmatrix} \tau_\Lambda & 0 \\ 0 & \tau_\Sigma \end{pmatrix} < g_1| \begin{pmatrix} G_0^\Lambda t^{\Lambda\Lambda} G_0^\Lambda & G_0^\Lambda t^{\Lambda\Sigma} G_0^\Sigma \\ G_0^\Sigma t^{\Sigma\Lambda} G_0^\Lambda & G_0^\Sigma t^{\Sigma\Sigma} G_0^\Sigma \end{pmatrix} |g_1 > \begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix} \\ & + \begin{pmatrix} \tau_\Lambda & 0 \\ 0 & \tau_\Sigma \end{pmatrix} < g_1| \begin{pmatrix} G_0^\Lambda + G_0^\Lambda t^{\Lambda\Lambda} G_0^\Lambda & G_0^\Lambda t^{\Lambda\Sigma} G_0^\Sigma \\ G_0^\Sigma t^{\Sigma\Lambda} G_0^\Lambda & G_0^\Sigma + G_0^\Sigma t^{\Sigma\Sigma} G_0^\Sigma \end{pmatrix} \begin{pmatrix} |g_2^\Lambda > \\ |g_2^\Sigma > \end{pmatrix} X_N, \end{aligned} \tag{29}$$

$$\begin{aligned} X_N = & \tau_N \left(< g_2^\Lambda| \;\; < g_2^\Sigma| \right) \begin{pmatrix} G_0^\Lambda + G_0^\Lambda t^{\Lambda\Lambda} G_0^\Lambda & G_0^\Lambda t^{\Lambda\Sigma} G_0^\Sigma \\ G_0^\Sigma t^{\Sigma\Lambda} G_0^\Lambda & G_0^\Sigma + G_0^\Sigma t^{\Sigma\Sigma} G_0^\Sigma \end{pmatrix} |g_1 > \begin{pmatrix} X_\Lambda \\ X_\Sigma \end{pmatrix} \\ & + \tau_N \left(< g_2^\Lambda| \;\; < g_2^\Sigma| \right) \begin{pmatrix} G_0^\Lambda t^{\Lambda\Lambda} G_0^\Lambda & G_0^\Lambda t^{\Lambda\Sigma} G_0^\Sigma \\ G_0^\Sigma t^{\Sigma\Lambda} G_0^\Lambda & G_0^\Sigma t^{\Sigma\Sigma} G_0^\Sigma \end{pmatrix} \begin{pmatrix} |g_2^\Lambda > \\ |g_2^\Sigma > \end{pmatrix} X_N, \end{aligned} \tag{30}$$

which are integral equations in one continuous variable given explicitly by

$$\begin{aligned} X_\alpha(q_1) = & \tau_\alpha(q_1) \sum_{\beta=\Lambda,\Sigma} \int_0^\infty {q_1'}^2 dq_1' K^{\alpha\beta}(q_1, q_1') X_\beta(q_1') \\ & + \tau_\alpha(q_1) \int_0^\infty q_2^2 dq_2 K^{\alpha N}(q_1, q_2) X_N(q_2); \qquad \alpha = \Lambda, \Sigma, \end{aligned} \tag{31}$$

$$\begin{aligned} X_N(q_2) = & \tau_N(q_2) \sum_{\alpha=\Lambda,\Sigma} \int_0^\infty q_1^2 dq_1 K^{N\alpha}(q_2, q_1) X_\alpha(q_1) \\ & + \tau_N(q_2) \int_0^\infty {q_2'}^2 dq_2' K^{NN}(q_2, q_2') X_N(q_2'). \end{aligned} \tag{32}$$

The kernels of these integral equations are given in the Appendix.

IV. RESULTS AND DISCUSSION

We applied the formalism of the previous section, using six different versions of the YN interaction obtained from the CQM, all of which reproduce equally well the experimental low-energy YN data [15, 16]. Results are listed in Table II from where it is clear that the $\pi\Lambda N$ binding energies are substantial for πY models with $A_2 < 1$ fm^2. Generally, the higher the r.m.s. momentum of the πY form factor g_2, the stronger is the binding, as demonstrated in the table. Irrespective of which πY model is chosen, the YN interaction always produces repulsion, thus lowering the calculated binding energy, as demonstrated by the results listed in the last line which corresponds to switching off the YN interaction. This repulsive YN effect owes its origin to the high-momentum components of the πB form factors which within the three-body calculation highlight the short-range repulsive region of the YN interaction.

To demonstrate the model dependence of the three-body calculation within a given πY model, we assembled in Table III several binding energy results based on model C and also on its limitation to the $\pi\Lambda$ channel (model F

TABLE II: Binding energy of $\pi\Lambda N$ (in MeV) for five πY interaction models (A-E of Table I) and six CQM versions of the $^3S_1-{}^3D_1$ YN interaction fitted to given ΛN scattering length a and effective range r_0 (both in fm). The momentum $p_{\rm lab}(\delta=0)$ is the Λ laboratory momentum (in MeV/c) where the $^3S_1$ ΛN phase shift changes sign. The last line corresponds to switching off the YN interaction.

a	r_0	$p_{\rm lab}(\delta=0)$	Model A	Model B	Model C	Model D	Model E
-1.35	3.39	987	147	99	65	30	6
-1.40	3.32	1011	147	99	66	30	6
-1.64	3.09	1146	150	102	68	32	8
-1.71	3.03	1198	150	102	68	33	9
-1.78	2.98	1272	151	103	69	33	9
-1.86	2.93	1446	152	104	69	34	10
–	–	–	170	120	84	47	21

TABLE III: Comparison of $\pi\Lambda N$ binding energies (in MeV) calculated within the πY model C and its $\pi\Lambda$ limit model F (see Table I) for YN coupled channels and ΛN single-channel models with $a=-1.40$ fm, and for no YN interaction.

$\pi\Lambda$, ΛN	$\pi\Lambda$, YN	πY, no YN	πY, ΛN	πY, YN
93	96	84	73	66

listed in Table I). The YN models included in this table invariably give $a=-1.40$ fm, whether limited to the ΛN $^3S_1$ single channel or extended to the $^3S_1-{}^3D_1$ $\Lambda N-\Sigma N$ coupled channels. Comparing the first two entries to each other, we conclude that the extension from a single ΛN channel to $\Lambda N-\Sigma N$ coupled channels has very little effect (about 3 MeV additional attraction) within the $\pi\Lambda$ model F. In contrast, for the full πY model C, as in the last two entries, the extension from ΛN to YN models has a somewhat larger effect (about 7 MeV repulsion) and in the opposite direction. Within the πY model C, the full coupled channels YN interaction contributes 18 MeV repulsion (third and fifth entries in Table III) to the three-body binding energy. Similar results hold for all other πY models.

To discuss the model dependence of the three-body calculation within a given YN CQM, we follow Ref. [14] in singling out $p_{\rm lab}(\delta=0)$, the momentum where the ΛN $^3S_1$ phase shift changes sign from attraction outside to repulsion inside, as a measure of the repulsive YN effect. We notice in Table II that the CQM values of $p_{\rm lab}(\delta=0)$ are considerably larger than those obtained by other models [20–22], signifying less repulsion in the CQM. In order to test whether the apparent lack of repulsion in the CQM YN interaction is responsible for the large binding energies obtained for $A_2<1$ fm^2, we added to the CQM with $a=-1.40$ fm and $r_0=3.32$ fm a short-range potential in the $^3S_1$ ΛN partial wave of the form

$$V(r)=\gamma_R\frac{e^{-\beta_R r}}{r}-\gamma_A\frac{e^{-\beta_A r}}{r}, \tag{33}$$

with $\beta_R=10$ fm^{-1} and $\gamma_R\geq 1000$ MeV fm, while the attractive term was adjusted to maintain the ΛN scattering length $a=-1.35$ fm and the effective range r_0 as close as possible to 3.39 fm, so that the YN observables are not changed noticeably. The overall effect of $V(r)$ is repulsive, as demonstrated in Table IV for the πY model A, with $A_2=0.8$ fm^2, where it is clearly seen that increase in the strength of the repulsive term lowers the value of $p_{\rm lab}(\delta=0)$ as well as lowering the $\pi\Lambda N$ binding energy.

TABLE IV: Binding energy of $\pi\Lambda N$ (in MeV) for the πY model A ($A_2 = 0.8$ fm^2) and the CQM YN interaction plus a short-range ΛN potential $V(r)$, Eq. (33), with scattering length $a = -1.40$ fm and effective range $r_0 = 3.32$ fm. The strength parameters γ are in units of MeV fm, the inverse range parameter β_A is in units of fm^{-1}, $\beta_R = 10$ fm^{-1}, and $p_{\rm lab}(\delta = 0)$ is the laboratory momentum (in MeV/c) where the $^3S_1$ ΛN phase shift changes sign.

γ_R	γ_A	β_A	$p_{\rm lab}(\delta = 0)$	$B(A_2 = 0.8)$
1000	240	5.371	873	107
2000	530	5.811	846	88
3000	720	5.749	822	70
4000	990	5.928	810	59
5000	1260	6.056	802	51
6000	1360	5.921	788	39
7000	1670	6.086	775	34

V. SUMMARY

In this work, we have extended the Faddeev equations study of a $(J^P, I) = (2^+, \frac{3}{2})$ quasibound $\pi\Lambda N$ state [14] from a $^3S_1$ ΛN single-channel to $^3S_1 - {}^3D_1$, $\Lambda N - \Sigma N$ coupled channels, and from a $\pi\Lambda$ single-channel description of $\Sigma(1385)$ to $\pi\Lambda - \pi\Sigma$ coupled channels description. Local interaction potentials given by the CQM were used in the YN sector, whereas one-rank separable potentials were used in the πB sectors. We have shown within a nonrelativistic version of the Faddeev equations, but using semirelativistic kinematics, that the $\pi\Lambda N$ system is bound under a wide choice of parametrizations of the πY interaction form factor. The form factors of the πB subsystems are sufficiently short ranged such that the pion undergoes almost coherently attraction to both baryons. The short-ranged repulsion between the two baryons in the CQM is insufficient to overcome the attraction gained by the pion unless the CQM is modified arbitrarily at very short distances to do this job. Altogether, the acceptable model dependence of the πY interaction form factor, and the uncertainty of the short-range behavior of the YN interaction, leave plenty of room, theoretically, for a quasibound $S = -1$, $(J^P, I) = (2^+, \frac{3}{2})$, $\pi\Lambda N$ dibaryon.

Before closing we list several production reactions, where the first two were already discussed in our previous paper [14], in which to search for this $S = -1$ dibaryon here denoted $\mathcal{D}$:

$$K^- + d \to \mathcal{D}^- + \pi^+ \ , \qquad \pi^- + d \to \mathcal{D}^- + K^+ \ , \tag{34}$$

$$p + p \to \mathcal{D}^+ + K^+ \ . \tag{35}$$

Correlated with the missing mass spectrum of the $\mathcal{D}$ dibaryon, for a forward outgoing meson, one should look for ΣN decays that can be assigned to a ΣN resonance with invariant mass $M_{\mathcal{D}}$. Total cross sections for the associated strangeness production $pp \to \Sigma N K^+$ near the hyperon production threshold have been reported from Juelich, for $\Sigma^0 p$ by the COSY-11 Collaboration [23], for $\Sigma^+ n$, also by COSY-11 [24], and by the ANKE Collaboration [25] and the HIRES Collaboration [26], with conflicting results among all these $\Sigma^+ n$ reports. Old DISTO data for the reaction $pp \to \Lambda p K^+$ have been analysed to search for an intermediate $K^- pp$ quasibound state, with the astounding report of a broad resonance at the $\pi\Sigma N$ threshold [27]. Of course, this $I = \frac{1}{2}$ resonance cannot be assigned to a $I = \frac{3}{2}$ $\pi\Lambda N$

quasibound state, but forthcoming data from the FOPI detector Collaboration at GSI [28] could be analysed also with respect to a ΣN rather than a Λp final state.

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Appendix: Expressions for the kernels of the integral equations Eqs. (31) and (32)

We provide here detailed expressions for the kernels appearing in the integral equations Eqs. (31) and (32).

$$\begin{aligned} K^{\alpha\beta}(q_1,q_1') = \; & \frac{1}{4}\sum_{\ell,\ell'=0,2}\int_0^\infty q_3^2 dq_3 \int_{-1}^{1} d\text{cos}\theta \int_{-1}^{1} d\text{cos}\theta' \\ & \times g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha,q_1) b_{13}^\alpha A_{13,1\alpha}^{10\ell 1}(q_1,q_3,\text{cos}\theta) \\ & \times t_{\ell\ell'}^{\alpha\beta}(p_3^\alpha, {p'}_3^\beta; E-q_3^2/2\nu_3) b_{31}^\beta A_{31,1\beta}^{\ell' 110}(q_3,q_1',\text{cos}\theta') G_0^\beta({p'}_1^\beta,q_1') g_1({p'}_1^\beta), \end{aligned} \tag{36}$$

$$\begin{aligned} K^{\alpha N}(q_1,q_2) = \; & \frac{1}{2}\int_{-1}^{1} d\text{cos}\theta\, g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha,q_1) b_{12}^\alpha A_{12,1\alpha}^{1010}(q_1,q_2,\text{cos}\theta) g_2^\alpha(p_2^\alpha) \\ & + \frac{1}{4}\sum_{\ell,\ell'=0,2}\sum_{\beta=\Lambda,\Sigma}\int_0^\infty q_3^2 dq_3 \int_{-1}^{1} d\text{cos}\theta \int_{-1}^{1} d\text{cos}\theta' \\ & \times g_1(p_1^\alpha) G_0^\alpha(p_1^\alpha,q_1) b_{13}^\alpha A_{13,1\alpha}^{10\ell 1}(q_1,q_3,\text{cos}\theta) \\ & \times t_{\ell\ell'}^{\alpha\beta}(p_3^\alpha, {p'}_3^\beta; E-q_3^2/2\nu_3) b_{32}^\beta A_{32,1\beta}^{\ell' 110}(q_3,q_2,\text{cos}\theta') G_0^\beta(p_2^\beta,q_2) g_2^\beta(p_2^\beta), \end{aligned} \tag{37}$$

$$K^{N\alpha}(q_2,q_1) = K^{\alpha N}(q_1,q_2), \tag{38}$$

$$\begin{aligned} K^{NN}(q_2,q_2') = \; & \frac{1}{4}\sum_{\ell,\ell'=0,2}\sum_{\alpha,\beta=\Lambda,\Sigma}\int_0^\infty q_3^2 dq_3 \int_{-1}^{1} d\text{cos}\theta \int_{-1}^{1} d\text{cos}\theta' \\ & \times g_2^\alpha(p_2^\alpha) G_0^\alpha(p_2^\alpha,q_2) b_{23}^\alpha A_{23,1\alpha}^{10\ell 1}(q_2,q_3,\text{cos}\theta) \\ & \times t_{\ell\ell'}^{\alpha\beta}(p_3^\alpha, {p'}_3^\beta; E-q_3^2/2\nu_3) b_{32}^\beta A_{32,1\beta}^{\ell' 110}(q_3,q_2',\text{cos}\theta') G_0^\beta({p'}_2^\beta,q_2') g_2^\beta({p'}_2^\beta), \end{aligned} \tag{39}$$

with

$$G_0^\alpha(p_i,q_i) = \frac{1}{E-\delta_{\alpha\Sigma}\Delta E - p_i^2/2\eta_i^\alpha - q_i^2/2\nu_i^\alpha + i\epsilon}, \qquad \alpha=\Lambda,\Sigma. \tag{40}$$

The orbital angular momentum recoupling coefficients $A_{ij,L\alpha}^{\ell_i\lambda_i\ell_j\lambda_j}(q_i,q_j,\text{cos}\theta) = A_{ji,L\alpha}^{\ell_j\lambda_j\ell_i\lambda_i}(q_j,q_i,\text{cos}\theta)$, and isospin recoupling coefficients $b_{ij}^\alpha = b_{ji}^\alpha$, are calculated by consideration of a cyclic pair ij. (The spin recoupling coefficients are all equal to 1.) For isospin we have

$$b_{ij}^{\alpha} = (-)^{i_j+\tau_j-I}\sqrt{(2i_i+1)(2i_j+1)}\,W(\tau_j\tau_k I\tau_i;i_i i_j), \tag{41}$$

where W is a Racah coefficient. If $\alpha = \Lambda$ then $\tau_1 = 0$, $\tau_2 = \frac{1}{2}$, $\tau_3 = 1$, $i_1 = \frac{3}{2}$, $i_2 = 1$, $i_3 = \frac{1}{2}$, and $I = \frac{3}{2}$ so that $b_{12}^{\Lambda} = b_{31}^{\Lambda} = b_{23}^{\Lambda} = 1$. If $\alpha = \Sigma$ we have instead $\tau_1 = 1$ so that $b_{12}^{\Sigma} = \sqrt{5/6}$, $b_{31}^{\Sigma} = -\sqrt{5}/3$, and $b_{23}^{\Sigma} = -1/\sqrt{6}$.

The orbital angular momentum recoupling coefficients are given by

$$\begin{aligned} A_{ij,L\alpha}^{\ell_i\lambda_i\ell_j\lambda_j}(q_i,q_j,\cos\theta) = &\ \frac{1}{2L+1}\sum_{Mm_im_j} C_{m_i,M-m_i,M}^{\ell_i\lambda_i L} C_{m_j,M-m_j,M}^{\ell_j\lambda_j L}\Gamma_{\ell_i m_i}\Gamma_{\lambda_i M-m_i} \\ &\times\Gamma_{\ell_j m_j}\Gamma_{\lambda_j M-m_j}\cos(-M\theta - m_i\theta_i^{\alpha} + m_j\theta_j^{\alpha}); \qquad \alpha = \Lambda,\Sigma, \end{aligned} \tag{42}$$

where $\Gamma_{\ell m} = 0$ for odd values of $\ell - m$, and

$$\Gamma_{\ell m} = \frac{(-1)^{(\ell+m)/2}\sqrt{(2\ell+1)(\ell+m)!(\ell-m)!}}{2^{\ell}[(\ell+m)/2]![(\ell-m)/2]!}, \tag{43}$$

for even values of $\ell - m$. The angles θ_i^{α} and θ_j^{α} are obtained from

$$\cos\theta_i^{\alpha} = -\frac{q_j\cos\theta + q_i a_{ij}^{\alpha}}{p_i^{\alpha}}, \tag{44}$$

$$\cos\theta_j^{\alpha} = \frac{q_i\cos\theta + q_j a_{ji}^{\alpha}}{p_j^{\alpha}}, \tag{45}$$

$$p_i^{\alpha} = \sqrt{q_j^2 + (q_i a_{ij}^{\alpha})^2 + 2q_iq_ja_{ij}^{\alpha}\cos\theta}, \tag{46}$$

$$p_j^{\alpha} = \sqrt{q_i^2 + (q_j a_{ji}^{\alpha})^2 + 2q_iq_ja_{ji}^{\alpha}\cos\theta}, \tag{47}$$

where

$$a_{ij}^{\alpha} = \frac{m_j}{m_j+m_k}, \qquad a_{ji}^{\alpha} = \frac{m_i}{m_i+m_k}, \tag{48}$$

with $m_1 = m_\alpha$; $\alpha = \Lambda,\Sigma$. Equations (46) and (47) provide also the relative momenta appearing in Eqs. (36)-(39).

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[1] M. Oka and K. Yazaki, Phys. Lett. **90B**, 41 (1980).

[2] M. Bashkanov *et al.* (CELSIUS/WASA Collaboration), Phys. Rev. Lett. **102**, 052301 (2009); see, however, T. Skorodko *et al.* (CELSIUS/WASA Collaboration), Phys. Lett. **B679**, 30 (2009), and S. Dymov *et al.* (COSY-ANKE Collaboration), Phys. Rev. Lett. **102**, 192301 (2009).

[3] R. L. Jaffe, Phys. Rev. Lett. **38**, 195 (1977).

[4] B. Bassalleck, Nucl. Phys. **A639**, 401c (1998); B. A. Cole, Nucl. Phys. **A639**, 407c (1998); H. J. Crawford, Nucl. Phys. **A639**, 417c (1998).

[5] T. Goldman, K. Maltman, G. J. Stephenson, Jr., K. E. Schmidt, and F. Wang, Phys. Rev. Lett. **59**, 627 (1987).

[6] M. Oka, K. Shimizu, and K. Yazaki, Phys. Lett. **130B**, 365 (1983).

[7] M. Oka, Phys. Rev. D **38**, 298 (1988).

[8] T. H. Tan, Phys. Rev. Lett. **23**, 395 (1969).

[9] G. Toker, A. Gal, and J. M. Eisenberg, Phys. Lett. **88B**, 235 (1979); Nucl. Phys. **A362**, 405 (1981).

[10] M. Torres, R. H. Dalitz, and A. Deloff, Phys. Lett. **B174**, 213 (1986).

[11] P. J. Mulders, A. T. Aerts, and J. J. de Swart, Phys. Rev. D **21**, 2653 (1980).

[12] K. Johnston *et al.*, Phys. Rev. C **46**, R1573 (1992).

[13] R. E. Chrien, C. B. Dover, and A. Gal, Czech. J. Phys. **42**, 1089 (1992).

[14] A. Gal and H. Garcilazo, Phys. Rev. D **78**, 014013 (2008).

[15] H. Garcilazo, T. Fernández-Carames, and A. Valcarce, Phys. Rev. C **75**, 034002 (2007).

[16] H. Garcilazo, A. Valcarce, and T. Fernández-Carames, Phys. Rev. C **76**, 034001 (2007).

[17] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **74**, 045205 (2006).

[18] C. Amsler *et al.*, Phys. Lett. **B667**, 1 (2008).

[19] A. Valcarce, H. Garcilazo, F. Fernández, and P. González, Rep. Prog. Phys. **68**, 965 (2005).

[20] Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, 21 (1999).

[21] Th. A. Rijken and Y. Yamamoto, Phys. Rev. C **73**, 044008 (2006).

[22] H. Polinder, J. Haidenbauer, and U.-G. Meissner, Nucl. Phys. **A779**, 244 (2006).

[23] P. Kowina *et al.*, Eur. Phys. J **A22**, 293 (2004).

[24] T. Rozek *et al.*, Phys. Lett. **B643**, 251 (2006).

[25] Yu. Valdau *et al.*, Phys. Lett. **B652**, 245 (2007); Phys. Rev. C **81**, 045208 (2010).

[26] A. Budzanowski *et al.* (HIRES Collaboration), submitted to Phys. Lett. **B** (15 March 2010).

[27] T. Yamazaki *et al.*, Phys. Rev. Lett. **104**, 132502 (2010).

[28] K. Suzuki, Nucl. Phys. **A827**, 312c (2009).